

On the supersymmetric completion of the R^4 term in M-theory

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Abstract

We examine the question of finding the supersymmetric completion of the R^4 term in M-theory. Using superfield methods, we present an eight derivative action in eight dimensions that has 32 preserved supersymmetries. We show also that this action has a hidden eleven-dimensional Lorentz invariance. It can thus be uplifted to give the complete set of bosonic terms in the M-theory eight derivative action.

1 Introduction

At low energies, string theory can be reduced to an effective field theory of the massless modes. The leading two-derivative action for these fields is the supergravity action S_2 . The effective action also contains an infinite series of higher derivative terms, suppressed by powers of the string scale α' , and the complete action has the form

$$S = S_2 + (\alpha')^4 S_8 + (\alpha')^5 S_{10} + \dots \quad (1.1)$$

where S_n contains terms with n derivatives. The leading correction in type II theories is the eight-derivative action, which contains the famous R^4 term [1, 2]

$$S_{8;R^4} = \int d^{10}x \, t^8 t^8 R^4 \quad (1.2)$$

Previous work on the eight derivative terms has produced many important results [3, 4, 5]. Most importantly, several nonrenormalization theorems are known which strongly restrict the moduli dependence of the eight-derivative terms. In particular it is known that in IIA theory, the R^4 term occurs only at tree-level and one-loop. The R^4 action in M-theory can then be obtained by taking the strong coupling limit of IIA theory [4].

There are also several other terms at the eight derivative level, which involve the other fields of the theory (in M-theory these fields are the gravitino and three-form field \hat{C}_{MNP}). These terms are believed to be related to the R^4 term by supersymmetry. However, little is known about the detailed structure of these terms.

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There are several reasons that one wishes to know the full action at the eight-derivative level.

At the basic level, knowledge of these terms will tell us a lot more about actions with maximal supersymmetry, which may lead to fundamental understandings like the off-shell nature of the theory.

From a phenomenological viewpoint, there has been a lot of interest in flux compactifications, where fluxes are turned on in the internal manifold (see e.g. [6]). This can apply both to the case of string theory on a Calabi-Yau manifold, or M-theory on a G_2 manifold. The potential for moduli in this background can be efficiently computed in the low energy effective theory, and can be used to gain information about stable compactifications at large radius. However, one needs to know the full action including all the field strengths. The full action may also be needed to consider the stabilization of the brane moduli.

Another place where the full effective action is required is for computing corrections in Anti-de-Sitter (AdS) backgrounds, for applications to the AdS/CFT correspondence (e.g. [7]). These can be applied to find corrections to black hole entropy, or to correlation functions.

Despite these motivations, it has not been possible so far to determine the complete eight derivative action. Several different approaches have been tried. All of these approaches have their own problems.

In string theory, the action can be computed by evaluating all the relevant string diagrams, and extracting the low energy action [1, 2, 8, 9]. In M-theory, a similar approach can be used using superparticle vertex operators [10]. The R^4 term can be found in this way. Alternatively, one can use sigma model techniques [11].

Unfortunately, string diagrams contain much more information than just the eight-derivative terms. One needs an effective way of extracting the low energy limit without doing the entire computation. Furthermore, once we get to five-point amplitudes and beyond, we have to worry about extracting contributions to the amplitude involving the exchange of massless fields, for example those coming from a combination of the four-point eight derivative amplitude and a tree level three-point interaction. Furthermore, the plethora of fields in the supergravities means that many amplitudes need to be computed. Sigma model techniques also require intense computational effort.

Another approach is to use the high supersymmetry of the theory. It is believed that the eight-derivative action is completely determined by supersymmetry alone. One can therefore attempt to construct the action by using the Noether method to generate terms step by step until supersymmetry is satisfied. This has been attempted for the heterotic string action [12, 13, 14, 15]. Unfortunately, the vast number of fields and the plethora of possible terms make it impractical to use this method directly in ten-dimensional supergravity, and even the eleven-dimensional case is very difficult.

The most promising approach is to use superfield methods. If the complete superfield can be found, then the action can be written as an integral over one-half of superspace. This has been attempted for the heterotic string in [16], and discussed for the maximally supersymmetric theories [17] (see also [18]). For the case of M-theory, this might seem hopeless, as there cannot be a chiral superfield in eleven dimensions (but see [19]).

In this paper, we shall show that the superfield approach can indeed be used to obtain the complete eight-derivative effective action in M-theory. This will require us to perform one trick: the requisite action is constructed in eight dimensions rather than eleven. That is, an action can be constructed which has all the required supersymmetry and manifest eight-dimensional Lorentz invariance. We then show that in fact there is a hidden eleven dimensional Lorentz invariance. Thus the eleven dimensional action can be straightforwardly found by a dimensional oxidation of this action to eleven dimensions.

The reason we need to go to eight dimensions is a natural consequence of the structure of superfield actions. The lowest component of a chiral superfield must be a complex scalar which does not exist in eleven dimensions. To get a complex scalar in a chiral theory, we need to dimensionally reduce to eight dimensions. Hence instead of trying to find the M-theory action directly, we will try to find the dimensional reduction of the action on a three-torus. The dimensionally reduced action will be constructed by superfield methods.

In fact, we do not even need to find the full action in the lower dimensional theory. For example, the eleven dimensional term of the form $R^2\hat{G}_4^2$ will yield, after dimensional reduction, terms like $R^2G_4^2$ as well as $R^2H_3^2$. If we can establish the exact form of either of these terms in eight dimensions, we can dimensionally oxidize to reproduce the eleven dimensional term.

The dimensional reduction of eleven-dimensional supergravity on a three-torus produces $N = 2, D = 8$ supergravity and was originally performed in [20]. The scalars from the reduction of the metric are the volume of the three-torus and 5 scalars L_i^m ($i, m = 1, 2, 3$). There is also the scalar \hat{C}_{123} . These 7 scalars parametrize a $SL(3, R)/SO(3, R) \times SL(2, R)/U(1)$ coset space. In addition, the theory contains three 2-form fields, 3 gauge fields, and a three-form field. The details of this reduction are worked out in the next section.

We then build a chiral superfield for this theory. The lowest component of this superfield is a complex scalar built out of C_{123} and the volume of the three-torus. The curvature occurs, as expected, with a coefficient of θ^4 . We can therefore expect to obtain a supersymmetric action by integrating the fourth power of the superfield over half of superspace.

Unfortunately, this is not the case. The reason is that we also need a supersymmetric measure; the supersymmetric analogue of the \sqrt{g} factor. Now it is not obvious that such a measure exists, and in fact, in the very similar situation of type IIB supergravity, it can be shown that such a measure does not exist [21, 22]. There is a similar obstruction in our case, and hence the supersymmetric action suggested above does not exist.

The way around this for type IIB was suggested in [23], and we shall apply the same reasoning here. Instead of trying to construct the full action in eight dimensions, we shall look for a subset of the terms.

Explicitly, we only consider bosonic terms which are composed out of the curvature R , the three-form field strengths $H_{\mu\nu\rho m}$, and the scalars L_m^i (each of these is uncharged under this $U(1)$ symmetry). Furthermore, we consider terms which are composed out of the bosonic terms listed above, and in addition contain two fermions of charge $1/2$ and $-1/2$ respectively.

We can now go ahead and fix these terms by requiring the cancellation of the variations.

This is tedious, but the superfield approach can help us fix these terms. Our crucial claim is that the superfield correctly enforces this cancellation; the superfield action will thus reproduce correctly the specific subset of the terms that we have described above. To substantiate this claim, we perform an explicit evaluation of the variations of the action, and explicitly show that the variations cancel (this calculation is very similar to the one performed in [22, 23]).

We can therefore use the superfield to produce an action involving the curvature R , the three-form field strengths $H_{\mu\nu\rho m}$, and the scalars L_m^i . This is sufficient, as we have mentioned, to recover the eleven dimensional action, as long as the eight-dimensional action has the form of a dimensionally reduced action, that is, it should have a hidden eleven dimensional Lorentz invariance.

We must therefore confirm that our action has this hidden Lorentz invariance. This can be done in a straightforward way, by summing over an entire orbit of terms generated by the eleven dimensional rotations. The resulting action then has eleven dimensional symmetry, and 32 supercharges. It can therefore be dimensionally oxidized to find the eleven-dimensional action.

We close with a discussion of future directions.

2 N=2, D=8 supergravity

$N = 2, D = 8$ supergravity can be obtained as a direct dimensional reduction of $N = 1, D = 11$ supergravity to eight dimensions. The bosonic sector of the theory contains 7 scalars, 6 vectors, 3 two-form fields, one three-form and a graviton. The fermion sector contains two gravitinos and four fermions. We perform the explicit dimensional reduction, following [20].

We denote the 11D fields by $\{\hat{e}_{\hat{\mu}}^{\hat{a}}, \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}}, \hat{\psi}_{\hat{\mu}}\}$ where hatted indices run from 0 to 10. Space-time indices are denoted $\hat{\mu}$ while tangent-space indices are denoted \hat{a} .

The eleven dimensional supersymmetry variations are taken to be

$$\delta \hat{e}_{\hat{\mu}}^{\hat{a}} = -\frac{i}{2} \bar{\epsilon} \Gamma^{\hat{a}} \hat{\psi}_{\hat{\mu}} \quad (2.3)$$

$$\delta \hat{C}_{\hat{\mu}\hat{\nu}\hat{\rho}} = \frac{3}{2} \bar{\epsilon} \Gamma_{[\hat{\mu}\hat{\nu}} \hat{\psi}_{\hat{\rho}]} \quad (2.4)$$

$$\delta \hat{\psi}_{\hat{\mu}} = 2 \hat{D}_{\hat{\mu}} \epsilon + \frac{i}{144} (\Gamma_{\hat{\mu}}^{\hat{a}\hat{b}\hat{c}\hat{d}} - 8 \delta_{\hat{\mu}}^{\hat{a}} \gamma^{\hat{b}\hat{c}\hat{d}}) \hat{F}_{\hat{a}\hat{b}\hat{c}\hat{d}} \quad (2.5)$$

We split the coordinates $x^{\hat{\mu}} = (x^{\mu}, z^m)$ with $\mu = (0, 1, \dots, 7)$ and $m = (1, 2, 3)$. Correspondingly, we split the indices $\hat{\mu} = (\mu, m)$, $\hat{a} = (a, i)$ where μ, a run from 0 to 7, and m, i run over 1, 2, 3. The bosonic fields are reduced via the ansatz

$$\hat{e}_{\hat{\mu}}^{\hat{a}} = \begin{pmatrix} e^{-\frac{1}{6}\varphi} e_{\mu}^a & e^{\frac{1}{3}\varphi} L_m^i A^m_{\mu} \\ 0 & e^{\frac{1}{3}\varphi} L_m^i \end{pmatrix} \quad (2.6)$$

and

$$\hat{C}_{abc} = e^{\frac{1}{2}\varphi} C_{abc}, \quad \hat{C}_{abi} = L_i^m B_{abm}, \quad \hat{C}_{aij} = e^{-\frac{1}{2}\varphi} L_i^m L_j^n V_{amn}, \quad \hat{C}_{ijk} = e^{-\varphi} \epsilon_{ijk} \ell. \quad (2.7)$$

The fermions are reduced by the ansatz

$$\hat{\psi}_{\hat{a}} = e^{\varphi/12} \left(\psi_a - \frac{1}{6} \Gamma_a \Gamma^i \lambda_i \right), \quad \hat{\psi}_i = e^{\varphi/12} \lambda_i, \quad \hat{\epsilon} = e^{-\varphi/12} \epsilon. \quad (2.8)$$

We also define

$$\mathcal{M}_{mn} = -L_m^i L_n^j \eta_{ij}, \quad (2.9)$$

where $\eta_{ij} = -I_3$ is the internal flat metric.

The dimensional reduction of the eleven dimensional field strength \hat{G} leads to the eight-dimensional field strengths

$$\begin{aligned} G_{\mu\nu\rho\lambda} &= 4\partial_{[\mu} C_{\nu\rho\lambda]} + 6F_{[\mu\nu}^m B_{\rho\lambda]m}, \\ G_{\mu\nu\rho i} &= L_i^m G_{\mu\nu\rho m} = L_i^m (3\mathcal{D}_{[\mu} B_{\nu\rho]m} + 3F_{[\mu\nu}^m V_{\rho]mn}) \\ G_{\mu\nu ij} &= L_i^m L_j^n G_{\mu\nu mn} = L_i^m L_j^n (2\mathcal{D}_{[\mu} V_{\nu]mn} + \ell \epsilon_{mnp} F_{\mu\nu}^p) \\ G_{\mu ijk} &= L_i^m L_j^n L_k^p G_{\mu mnp} = L_i^m L_j^n L_k^p \epsilon_{mnp} \partial_\mu \ell \end{aligned} \quad (2.10)$$

where the field strength of the gauge field is given by

$$F_{\mu\nu}^m = 2\partial_{[\mu} A_{\nu]}^m \quad (2.11)$$

The supersymmetry transformation rules in eight dimensions are

$$\delta e_\mu^a = -\frac{i}{2} \bar{\epsilon} \Gamma^a \psi_\mu \quad (2.12)$$

$$\begin{aligned} \delta \psi_\mu &= 2\partial_\mu \epsilon - \frac{1}{2} \omega_\mu^{ab} \Gamma_{ab} \epsilon + \frac{1}{2} L_i^m \mathcal{D}_\mu L_{mj} \Gamma^{ij} \epsilon + \frac{i}{96} e^{\varphi/2} (\Gamma_\mu^{\nu\rho\delta\epsilon} - 4\delta_\mu^\nu \Gamma^{\rho\delta\epsilon}) G_{\nu\rho\delta\epsilon} \epsilon \\ &\quad - \frac{i}{12} e^{-\varphi} \Gamma^{ijk} G_{\mu ijk} \epsilon + \frac{1}{24} e^{\varphi/2} \Gamma^i L_i^m (\Gamma_\mu^{\nu\rho} - 10\delta_\mu^\nu \Gamma^\rho) F_{m\nu\rho} \epsilon \\ &\quad + \frac{i}{36} \Gamma^i (\Gamma_\mu^{\nu\rho\delta} - 6\delta_\mu^\nu \Gamma^{\rho\delta}) G_{\nu\rho\delta i} \epsilon + \frac{i}{48} e^{-\varphi/2} \Gamma^{ij} (\Gamma_\mu^{\nu\rho} - 10\delta_\mu^\nu \Gamma^\rho) G_{\nu\rho ij} \epsilon \end{aligned} \quad (2.13)$$

$$\begin{aligned} \delta \psi_i &= \frac{1}{2} L_i^m L^{jn} \mathcal{D} \mathcal{M}_{mn} \Gamma_j \epsilon - \frac{1}{3} \Gamma^\mu \partial_\mu \varphi \Gamma_i \epsilon + \frac{i}{24} e^{-\varphi/2} \Gamma^j (3\delta_i^k - \Gamma_i^k) \Gamma^{\mu\nu} G_{\mu\nu jk} \epsilon \\ &\quad - \frac{1}{4} e^{\varphi/2} L_i^m \mathcal{M}_{mn} \Gamma^{\mu\nu} F_{\mu\nu}^n \epsilon + \frac{i}{144} e^{\varphi/2} \Gamma_i \Gamma^{\mu\nu\rho\delta} G_{\mu\nu\rho\delta} \epsilon + \frac{i}{6} e^{-\varphi} \Gamma^{jk} \Gamma^\mu G_{\mu ijk} \epsilon \\ &\quad + \frac{i}{36} (2\delta_i^j - \Gamma_i^j) \Gamma^{\mu\nu\rho} G_{\mu\nu\rho j} \epsilon \end{aligned} \quad (2.14)$$

$$\delta A_\mu^m = -\frac{i}{2} e^{-\varphi/2} L_i^m \bar{\epsilon} (\Gamma^i \psi_\mu + \Gamma_\mu (\eta^{ij} + \frac{1}{6} \Gamma^i \Gamma^j) \lambda_j) \quad (2.15)$$

$$\delta V_{\mu mn} = \epsilon_{mnp} \left[-\frac{i}{2} e^{\varphi/2} L_i^p \bar{\epsilon} \Gamma^9 (\Gamma^i \psi_\mu + \Gamma_\mu (\eta^{ij} - \frac{5}{6} \Gamma^i \Gamma^j) \lambda_j) - \ell \delta A_\mu^p \right] \quad (2.16)$$

$$\delta B_{\mu\nu m} = L_m^i \bar{\epsilon} (\Gamma_{[\mu} \psi_{\nu]} + \frac{1}{6} \Gamma_{\mu\nu} (3\delta_i^j - \Gamma_i^j) \lambda_j) - 2\delta A_{[\mu}^n V_{\nu]mn} \quad (2.17)$$

$$\delta C_{\mu\nu\rho} = \frac{3}{2} e^{-\varphi/2} \bar{\epsilon} \Gamma_{[\mu\nu} (\psi_{\rho]} - \frac{1}{6} \Gamma_{\rho]} \Gamma^i \lambda_i) - 3\delta A_{[\mu}^m B_{\nu\rho]m} \quad (2.18)$$

$$\delta \varphi = -\frac{i}{2} \bar{\epsilon} \Gamma^i \lambda_i \quad (2.19)$$

$$\delta \ell = -\frac{i}{2} e^{\varphi} \bar{\epsilon} \Gamma^9 \Gamma^i \lambda_i \quad (2.20)$$

Here we have defined $\Gamma^9 = i\Gamma^{123}$.

This theory has a manifest $SL(3, R)$ acting on the compactification three-torus. There is also a $SL(2, R)$ symmetry, which corresponds to the electric-magnetic duality of 11-dimensional supergravity. We will now rewrite the fields to make this more manifest.

To represent the $SL(2, R)$ symmetry linearly on the scalars, we must introduce an extra compensating scalar ϕ . The scalars are organized into a $SL(2, R)$ matrix

$$V = \frac{1}{\sqrt{2i}} \begin{pmatrix} u & u^* \\ v & v^* \end{pmatrix} \equiv \frac{1}{\sqrt{2i\tau_2}} \begin{pmatrix} \bar{\tau}e^{-i\phi} & \tau e^{i\phi} \\ e^{-i\phi} & e^{i\phi} \end{pmatrix} \quad (2.21)$$

Here $\tau = l + ie^\phi$ parametrizes the upper half plane. There is now a local $U(1)$ action that acts as a shift on the angular variable ϕ , and which can be used to set $\phi = 0$.

We will define $V_{\mu mn} = \epsilon_{mnp} W_\mu^p$. Then the potentials (A, W) form a $SL(2, R)$ doublet. They can be organized into $SL(2, R)$ invariant fields defined by

$$(a_\mu^m, (a_\mu^m)^*) = \sqrt{2i}(A_\mu^m, W_\mu^m)V \quad (2.22)$$

with the corresponding field strengths $f_{\mu\nu} = 2\partial_{[\mu}A_{\nu]}^m$.

Similarly, the four-form field strength G_{mnpq} and its dual four-form \tilde{G}_{mnpq} can be organized into $SL(2, R)$ invariant field strengths by

$$(F_{\mu\nu\rho\lambda}, F_{\mu\nu\rho\lambda}^*) = \sqrt{2i}(G_{\mu\nu\rho\lambda}, \tilde{G}_{\mu\nu\rho\lambda})V \quad (2.23)$$

In the fermion sector, we define

$$\begin{aligned} \zeta &= \frac{(1 + i\Gamma^9)}{2}\epsilon & \tilde{\psi}_\mu &= \frac{(1 + i\Gamma^9)}{2}\psi_\mu & \lambda &= \frac{(1 + i\Gamma^9)}{2}\Gamma^i\psi_i \\ \chi_m &= \frac{(1 - i\Gamma^9)}{2}(\psi_m - \frac{1}{3}\gamma_m\gamma^i\psi_i) \end{aligned} \quad (2.24)$$

Now we can rewrite the supersymmetry transformation laws

$$\delta e_\mu^a = -\frac{i}{2} \left(\zeta \Gamma^0 \Gamma^a \tilde{\psi}_\mu^* + \zeta^* \Gamma^0 \Gamma^a \tilde{\psi}_\mu \right) \quad (2.25)$$

$$\begin{aligned} \delta \tilde{\psi}_\mu &= 2\nabla_\mu \zeta + \frac{1}{2} L_i^m \mathcal{D}_\mu L_{mj} \Gamma^{ij} \zeta + \frac{i}{24} \Gamma^i L_i^m (\Gamma_\mu^{\nu\rho} - 10\delta_\mu^{\nu\rho}) f_{m\nu\rho}^* \zeta^* \\ &\quad - \frac{1}{192} \Gamma^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma}^* \gamma_\mu \zeta^* + \frac{i}{36} \Gamma^i L_i^m (\Gamma_\mu^{\nu\rho\delta} - 6\delta_\mu^{\nu\rho\delta}) G_{\nu\rho\delta m} \zeta \end{aligned} \quad (2.26)$$

$$\delta \lambda = \gamma^a p_a \zeta^* - \frac{i}{4} \Gamma^i L_i^m \Gamma^{\mu\nu} f_{\mu\nu m} \zeta + \frac{1}{96} \Gamma^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \zeta \quad (2.27)$$

$$\begin{aligned} \delta \chi_i &= \frac{1}{2} L_i^m L^{jn} \Gamma^\mu \mathcal{D}_\mu \mathcal{M}_{mn} \Gamma_j \zeta - \frac{i}{12} (3\delta_i^j - \gamma_i \gamma^j) L_j^p f_{p\mu\nu}^* \gamma^{\mu\nu} \zeta^* \\ &\quad + \frac{i}{36} (3\delta_i^j - \Gamma_i \Gamma^j) L_j^m \Gamma^{\mu\nu\rho} G_{\mu\nu\rho m} \zeta \end{aligned} \quad (2.28)$$

$$\delta a_\mu^m = -L_i^m \left(\zeta \Gamma^0 \Gamma^i \tilde{\psi}_\mu + \zeta^* \Gamma^0 \Gamma_\mu \chi^i + \frac{1}{2} \zeta^* \Gamma^0 \Gamma_\mu \Gamma^i \lambda \right) \quad (2.29)$$

$$\begin{aligned} \delta B_{\mu\nu m} &= L_m^i \zeta \Gamma^0 (\Gamma_{i[\mu} \tilde{\psi}_{\nu]}^* + \frac{1}{2} \Gamma_{\mu\nu} \chi_i^*) + L_m^i \zeta^* \Gamma^0 (\Gamma_{i[\mu} \tilde{\psi}_{\nu]} + \frac{1}{2} \Gamma_{\mu\nu} \chi_i) \\ &\quad + i\epsilon_{mnp} (a_\mu^n \delta a_\nu^{*p} - a_\mu^{*n} \delta a_\nu^p) \end{aligned} \quad (2.30)$$

$$\delta u = -\frac{1}{2iv} \zeta \Gamma^0 \lambda \quad (2.31)$$

The various field strengths are now charged under the local U(1) symmetry. The scalars u, v have charge 1. The field strengths f_{mn} and \hat{F} have charge 1, while the two-form field $B_{\mu\nu}$ and the curvature have zero charge. The gravitino $\tilde{\psi}_\mu$ and the fermion χ_m have charge 1/2, while the fermion λ has a charge 3/2.

3 The linearized superfield

We now start the superfield analysis of the theory.

The superspace coordinates are $(x^\mu, \theta^\alpha, \theta^{*\bar{\alpha}})$, where $\theta^{*\bar{\alpha}} = (\theta^\alpha)^*$, and θ is a 16 component Weyl spinor satisfying $(1 + i\Gamma^9)\theta = 0$. The supersymmetric derivatives are defined as

$$D_\alpha^j = \frac{\partial}{\partial \theta^\alpha} + i\theta^{*\bar{\alpha}} \Gamma_{\alpha\bar{\alpha}}^\mu \partial_\mu \quad \bar{D}_{\bar{\alpha}} = \frac{\partial}{\partial \theta^{*\bar{\alpha}}} + i\theta^\alpha \Gamma_{\alpha\bar{\alpha}}^\mu \partial_\mu \quad (3.32)$$

The chiral superfield satisfies $\bar{D}\Phi = 0$, and has for its lowest component the scalar u . The rest of the superfield can be determined from the basic equation for the supersymmetry variation of any superfield V

$$\delta_\zeta V = \zeta^\alpha D_\alpha V - \zeta^{*\bar{\alpha}} \bar{D}_{\bar{\alpha}} V \quad (3.33)$$

Repeatedly applying this equation, we find the components of the chiral superfield.

We find for the first few components at the linearized level

$$\Phi|_{\theta=0} = u \quad (3.34)$$

$$D_\alpha \Phi|_{\theta=0} = \frac{1}{2iv} (\Gamma^0 \lambda)_\alpha \quad (3.35)$$

$$D_{[\alpha} D_{\beta]} \Phi|_{\theta=0} = -\frac{1}{2iv} \left(\frac{i}{4} \Gamma^i L_i^m \Gamma^{\mu\nu} \Gamma^0 f_{\mu\nu m} + \frac{1}{96} \Gamma^{\mu\nu\rho\sigma} \Gamma^0 F_{\mu\nu\rho\sigma} \right)_{\beta\alpha} \quad (3.36)$$

$$D_{[\alpha} D_\beta D_{\gamma]} \Phi|_{\theta=0} = \frac{1}{8v} \left[(\Gamma^i \Gamma^{\mu\nu} \Gamma^0)_{\gamma\beta} \left((\Gamma^0 \Gamma^i)_{\alpha\delta} \tilde{\psi}_{\mu\nu}^\delta - 2(\Gamma^0 \Gamma_{[\mu})_{\alpha\delta} \partial_{\nu]} \chi^{i\delta} \right) + \frac{1}{4} (\Gamma^{\mu\nu\rho\lambda} \Gamma^0)_{\gamma\beta} (\Gamma^0 \Gamma_{[\mu\nu})_{\alpha\delta} \tilde{\psi}_{\lambda\rho]}^\delta \right] \quad (3.37)$$

We now work out the terms in the next order of the superfield which are proportional to the curvature. These terms are

$$D_{[\alpha} D_\beta D_\gamma D_{\delta]} \Phi|_{\theta=0} = -\frac{1}{16v} \left[(\Gamma^i \Gamma^{\lambda\rho} \Gamma^0)_{\delta\gamma} (\Gamma^0 \Gamma^i \Gamma_{\sigma\tau})_{\beta\alpha} + \frac{1}{4} (\Gamma^{\mu\nu\rho\lambda} \Gamma^0)_{\delta\gamma} (\Gamma^0 \Gamma_{\mu\nu\sigma\tau})_{\beta\alpha} \right] R_{\lambda\rho}^{\sigma\tau} \quad (3.38)$$

The terms in the superfield multiplied by θ^4 all have two derivatives. Thus if we integrate Φ^4 over half of superspace, we will produce an eight derivative action

$$S_8 = \int d^{10}x d^{16}\theta \Phi^4 \quad (3.39)$$

which will have linearized supersymmetry.

When we include the moduli, we can have a moduli-dependent coefficient multiplying this action; this coefficient is not itself determined by linearized supersymmetry. Explicit

computations in string theory show that the above action must be multiplied by the function $\ln(\eta(u))$. Hence the action would have the form (up to an overall constant)

$$S_8 = \ln(\eta(u)) \int d^{10}x d^{16}\theta \Phi^4 \quad (3.40)$$

4 The Nonlinear Action

We can now try and extend this to the nonlinear case.

When we try to go beyond the quartic action, we will need the full nonlinear superfield. In addition we need a supersymmetric measure; the supersymmetric analogue of the \sqrt{g} factor. The suggested form of the eight-derivative action is then

$$\begin{aligned} S_8 &= \int d^{10}x \int d^{16}\theta \Delta \Phi^4 \\ &= \int d^{10}x \epsilon^{\alpha_1 \dots \alpha_{16}} \sum_{n=0}^{16} \frac{1}{n!(16-n)!} D_{\alpha_1} \dots D_{\alpha_n} \Delta | D_{\alpha_{n+1}} \dots D_{\alpha_{16}} W | \end{aligned} \quad (4.41)$$

where $W = \Phi^4$, and Δ is by definition a superfield whose lowest component is

$$\Delta|_{\theta=0} = \sqrt{g} \quad (4.42)$$

Δ is to be constructed order by order by requiring that the action be supersymmetric.

Now it is not obvious that such a measure exists, and in fact, in the very similar situation of type IIB supergravity, it can be shown that such a measure does not exist [21, 22]. The issue is that while we can arrange that all variations proportional to ζ cancel, the variations proportional to ζ^* will then not cancel. There is a similar obstruction in this case, and hence the supersymmetric action suggested above does not exist.

In [23], it was shown that despite this problem, there was still some nontrivial information available from the superfield expression. In particular, a subset of the terms in the action is correctly generated from the superfield. The same reasoning will apply here.

To make this explicit, we now define the subset of the terms that we will look at.

We restrict attention to the bosonic terms which involve only the field strengths which are uncharged under the $U(1)$, viz. the curvature $R_{\mu\nu\rho\sigma}$, the three-form field strengths $H_{\mu\nu\rho m}$, and the scalars L_m^i . Examples of such terms are R^4 , $R^2 H^4$ etc.

Now under a supersymmetry transformation, these terms produce variations which contain one or more fermion fields; for instance, the variation of the R^4 term will produce variations of the generic form $R^3 \bar{\zeta} D^2 \psi$. This must be cancelled by the variation of terms bilinear in fermions, for example, a term of the form $R^2 D\psi D^2 \psi$. An analysis of the $U(1)$ structure shows that these terms must be of a particular form: they involve the uncharged fields $R_{\mu\nu\rho\sigma}$, $H_{\mu\nu\rho m}$, and L_m^i , and in addition they have two fermions, one of which carries a $1/2$ charge under the $U(1)$ (i.e. ψ_μ or χ_a), and one with a $-1/2$ charge under the $U(1)$ (i.e. ψ_μ^* or χ_a^*).

We fix these terms by requiring a cancellation of the variations. It will suffice to consider those variations which have at most one fermion field i.e. we ignore the cancellation of the terms with three fermions. The cancellation of variations with one fermion will be enough to determine the subset of bosonic terms in the action that we are considering.

Our crucial claim is that the superfield correctly enforces this cancellation; the superfield action (4.41) will thus reproduce correctly the specific subset of the terms that we have described above.

To prove this, we start by noting that the uncharged field strengths are all found in the θ^4 component of the superfield. The fermionic terms that we are considering are all to be found in the θ^3, θ^5 components. Thus, when we look for the bosonic terms in the action, the 16 θ are then already saturated from the Φ^4 term. For the terms bilinear in fermions, at least 15 θ must be taken from the Φ^4 term (as opposed to factors of θ coming from Δ).

Hence to construct the action, we only need the first two components of Δ , i.e. $\Delta|_{\theta=0} \equiv \sqrt{g}$ and $D_\alpha \Delta|_{\theta=0}$. We do not need the other components of the measure, as long as we are restricting ourselves to this particular subset of terms.

To summarize, we are setting $\partial\tau = \lambda_i = a_\mu^m = C_{\mu\nu\rho} = 0$, and we are considering variations with at most one fermion field. We may truncate the action to

$$S = \int d^8x \frac{1}{16!} \epsilon^{\alpha_1 \dots \alpha_{16}} (\sqrt{g} D_{\alpha_1} \dots D_{\alpha_{16}} W| + 16 D_{\alpha_1} \Delta| D_{\alpha_2} \dots D_{\alpha_{16}} W|) \quad (4.43)$$

We now need to show that this action is supersymmetric, and we shall do this in the next section. This analysis will follow [22] closely.

5 Cancelling the Supersymmetry Variations

To analyze the supersymmetry variations, we will need some facts about the torsions. These are determined by the algebra

$$[D_A, D_B] = -T_{AB}{}^C D_C + \frac{1}{2} R_{ABC}{}^D L_D{}^C + 2i M_{AB} \kappa, \quad (5.44)$$

We can set some torsions and curvatures to zero because there are no terms of the right dimension and charge. We then find that the nonzero torsions are $T_{\alpha\beta}{}^{\tilde{\gamma}}, T_{\alpha\beta}{}^c, T_{a\beta}{}^\gamma, T_{a\beta}{}^{\tilde{\gamma}}, T_{ab}{}^\gamma$ and their complex conjugates.

The curvatures are determined from the torsions by the Bianchi identities

$$\sum_{(ABC)} (D_A T_{BC}{}^D + T_{AB}{}^E T_{EC}{}^D - \hat{R}_{ABC}{}^D) = 0 \quad (5.45)$$

in particular

$$T_{\bar{\alpha}\beta}{}^c T_{c\gamma}{}^\delta + T_{\bar{\alpha}\gamma}{}^c T_{c\beta}{}^\delta + T_{\beta\gamma}{}^{\bar{e}} T_{\bar{e}\bar{\alpha}}{}^\delta - \hat{R}_{\bar{\alpha}\beta\gamma}{}^\delta = 0 \quad (5.46)$$

The torsions can be determined from the supersymmetry algebra. For example, we have

$$D_a D_\beta V - D_\beta D_a V = -T_{a\beta}{}^\gamma D_\gamma V \quad (5.47)$$

Noting that

$$D_a \equiv e_a^M D_M = e_a^m D_m - \frac{1}{2} \psi_a^\alpha D_\alpha + \frac{1}{2} \psi_a^{*\bar{\alpha}} D_{\bar{\alpha}} \quad (5.48)$$

we find that the algebra implies that

$$T_{\beta\bar{\alpha}}{}^c = -i(\Gamma^0 \Gamma^c)_{\beta\bar{\alpha}} \quad (5.49)$$

and

$$T_{a\beta}{}^\alpha = -\frac{1}{4}L_i{}^m\mathcal{D}_\mu L_{mj}(\Gamma^{ij})_{\alpha\beta} - \frac{i}{72}(\Gamma^i L_i{}^m(\Gamma_\mu{}^{\nu\rho\delta} - 6\delta_\mu{}^\nu\Gamma^{\rho\delta}))_{\alpha\beta}G_{\nu\rho\delta m} \quad (5.50)$$

Now we return to the considerations of the supersymmetry variations. Once again, we are setting $\partial\tau = \lambda = a_2 = 0$, and we are considering variations with at most one fermion field. We can then set $D^n W| = 0$ in the supersymmetry variations for all $n \leq 14$. We thus only need to cancel the variations proportional to $D^{16}W$ and $D^{15}W$. Furthermore we can set $[D_{\alpha_1}, D_{\alpha_2}]\Delta| = 0$, since it has a $U(1)$ charge of 1.

The variations are then

$$\delta S = \int d^{10}x \left[\delta e D^{16}W| + e(\delta D^{16}W|) + eD_\alpha\Delta|\delta D^{15,\alpha}W| + e\delta D_\alpha\Delta|D^{15,\alpha}W| \right] \quad (5.51)$$

Consider each term separately.

For the first term, the variation of e is

$$\delta e = -\frac{i}{2}ee^\mu{}_a(\zeta\Gamma^0\Gamma^a\psi_\mu^* + \zeta^*\Gamma^0\Gamma^a\psi_\mu) \quad (5.52)$$

In the second term, the variation of the $D^{16}W|$ term is

$$\delta D^{16}W| = \frac{1}{16!}\epsilon^{\alpha_1\dots\alpha_{16}}(\zeta^\alpha D_\alpha - \zeta^{*\bar{\alpha}}\bar{D}_{\bar{\alpha}})D_{\alpha_1}\dots D_{\alpha_{16}}W|. \quad (5.53)$$

In the ζ terms we can antisymmetrize the D_α derivatives, and since there are only 16 D_α , this term is zero. For the ζ^* terms, we compute the commutator $[\bar{D}_\alpha, D^{16}]W|$. We find

$$\delta D^{16}W| = -\zeta^{*\bar{\alpha}}\left(\frac{1}{2}T_{\bar{\alpha}\delta}{}^c\psi_c^\delta D^{16}W|\right) + \zeta^{*\bar{\alpha}}\left(e_c^m T_{\bar{\alpha}\beta}^c D_m + T_{\bar{\alpha}\gamma}^c T_{\beta c}^\gamma\right) D^{\beta,15}W|$$

where we have used (5.46), and dropped the torsions with $U(1)$ charge greater than $1/2$.

In the third term

$$\frac{1}{15!}\delta D^{15,\alpha}W = \frac{1}{15!}\epsilon^{\alpha\alpha_2\dots\alpha_{16}}(\zeta^\beta D_\beta - \zeta^{*\bar{\beta}}\bar{D}_{\bar{\beta}})(D_{\alpha_2}\dots D_{\alpha_{16}}W|) \quad (5.54)$$

The ζ^* terms are all of the form $D^n W$ with $n < 15$, and can be ignored. So

$$\frac{1}{15!}\delta D^{15,\alpha}W = \zeta^\alpha D^{16}W| \quad (5.55)$$

We can now calculate the total coefficient of $D^{16}W$. This is

$$\zeta^\alpha \left(-\frac{i}{2}ee_\mu^a\Gamma^0\Gamma^a\psi_\mu^* - eD_\alpha\Delta| \right) + \zeta^{*\bar{\alpha}} \left(-\frac{i}{2}ee_\mu^a(\Gamma^0\Gamma^a)_{\bar{\alpha}\beta}\psi_\mu^\beta - e\frac{1}{2}T_{\bar{\alpha}\delta}{}^c\psi_c^\delta \right) \quad (5.56)$$

The second term cancels. From the first term, we learn that we must take

$$D_\beta\Delta| = -\frac{i}{2}(\Gamma^0\Gamma^a)_{\beta\bar{\alpha}}\psi_a^{*\bar{\alpha}} = \frac{1}{2}T_{\beta\bar{\alpha}}^a\psi_a^{*\bar{\alpha}} \quad (5.57)$$

The coefficient of $D^{\beta,15}W|$ in the variation is then

$$\frac{1}{2}T_{\beta\bar{\alpha}}^a\delta\psi_{a\bar{\alpha}}^* + \zeta^{*\alpha} \left(e_c^m T_{\bar{\alpha}\beta}^c D_m + T_{\bar{\alpha}\gamma}^c T_{\beta c}^\gamma \right)$$

When taking the variation of the gravitino, the terms proportional to ζ all multiply terms with $U(1)$ charge greater than $1/2$, and can be dropped. The terms proportional to ζ^* are easily shown to cancel in the above coefficient up to a total derivative.

To summarize, we have shown here that the action (4.43) is invariant under supersymmetry transformations, after we perform the truncation described in the previous section.

6 Lorentz Invariance

Let us review what we have found so far. We have found supersymmetric actions of the form (4.43). In these expressions, Φ is a chiral superfield, but we only need the $\theta^{3,4,5}$ terms i.e. the superfield Φ can be truncated to $\Phi \sim \theta^3\Phi_3 + \theta^4\Phi_4 + \theta^5\Phi_5$. Then (4.43) will provide a supersymmetric expression as long as Φ_3 is a linear combination of terms which have $U(1)$ charge $1/2$. In our case, there are two fermions $\tilde{\psi}_{\mu\nu}$ and $\partial_\mu\chi_i$ which have this charge, and the proper dimension, and so Φ_3 in general can be taken to be a linear combination of these objects.

The correct linear combination, which we denote Φ_{inv} , can be determined by requiring the action to have eleven-dimensional Lorentz invariance.

Let us suppose we want to extend an $SO(2)$ invariant object to an $SO(3)$ invariant object. For example, take the $SO(2)$ invariant $A_x B_x + A_y B_y \equiv \sum_{i=1,2} A_i B_i$. Then the $SO(3)$ invariant object is immediately found to be $\sum_{i=1,3} A_i B_i$, that is, we simply extend the sum over all possible indices. The same principle can be applied to our case.

Now in our case, the third term in the superfield contains the term

$$\theta^\gamma \theta^\beta \theta^\alpha D_{[\alpha} D_\beta D_{\gamma]} \Phi|_{\theta=0} = \dots + \frac{i}{4} \bar{\theta} \Gamma^{\mu\nu\rho\lambda} \theta \bar{\theta} \Gamma_{[\mu\nu} \tilde{\psi}_{\lambda\rho]} \quad (6.58)$$

Here μ, ν, \dots run from 0 to 7. To make this Lorentz invariant, we should extend the sum over the eleven dimensional indices. For the gamma matrices, for instance, we must add terms where Γ^μ has been replaced with Γ^i .

For the gravitinos, we should use the relation between the eleven dimensional gravitino and the eight-dimensional gravitino

$$\hat{\psi}_{\hat{a}} = e^{\varphi/12} \left(\psi_a - \frac{1}{6} \Gamma_a \Gamma^i \lambda_i \right) \quad (6.59)$$

Now we are setting $\lambda_i = 0$ in all terms. We are also ignoring the moduli dependence. At this level of approximation, we can write the above term as

$$\theta^\gamma \theta^\beta \theta^\alpha D_{[\alpha} D_\beta D_{\gamma]} \Phi|_{\theta=0} = \dots + \frac{i}{4} \bar{\theta} \Gamma^{\mu\nu\rho\lambda} \theta \bar{\theta} \Gamma_{[\mu\nu} \hat{\psi}_{\lambda\rho]} \quad (6.60)$$

The extension of the term to a Lorentz invariant form is now straightforward; we thus get the Lorentz invariant object

$$W_3 = \frac{i}{4} \left(\bar{\theta} \Gamma^{\mu\nu\rho\lambda} \theta \bar{\theta} \Gamma_{\mu\nu} \hat{\psi}_{\lambda\rho} + \bar{\theta} \Gamma^{ij\rho\lambda} \theta \bar{\theta} \Gamma_{ij} \hat{\psi}_{\lambda\rho} + 4 \bar{\theta} \Gamma^{i\nu j\lambda} \theta \bar{\theta} \Gamma_{i\nu} \hat{\psi}_{j\lambda} + \bar{\theta} \Gamma^{\mu\nu ij} \theta \bar{\theta} \Gamma_{\mu\nu} \hat{\psi}_{ij} \right) \quad (6.61)$$

We have used the fact that θ is a Weyl spinor to simplify the expression.

This is the θ^3 component of the required Lorentz invariant superfield Φ_{inv} . To construct the action, we also need the θ^4, θ^5 components of Φ_{inv} . These can be found by using the

standard formula (3.33). We shall leave the explicit evaluation of these terms to a future paper, and here we will summarize these terms by formally replacing W_3 by the expression

$$W = \frac{i}{4} \left(\bar{\theta} \Gamma^{\mu\nu\rho\lambda} \theta \bar{\theta} \Gamma_{\mu\nu} \tilde{\Psi}_{\lambda\rho} + \bar{\theta} \Gamma^{ij\rho\lambda} \theta \bar{\theta} \Gamma_{ij} \tilde{\Psi}_{\lambda\rho} + 4 \bar{\theta} \Gamma^{i\nu j\lambda} \theta \bar{\theta} \Gamma_{i\nu} \tilde{\Psi}_{j\lambda} + \bar{\theta} \Gamma^{\mu\nu ij} \theta \bar{\theta} \Gamma_{\mu\nu} \tilde{\Psi}_{ij} \right) \quad (6.62)$$

We have here defined the new superfields $\tilde{\Psi}_{\lambda\rho}, \tilde{\Psi}_{j\lambda}, \tilde{\Psi}_{ij}$. The lowest components of these superfields are respectively $\hat{\psi}_{\lambda\rho}, \hat{\psi}_{j\lambda}, \hat{\psi}_{ij}$. (W itself is not a superfield; it should be thought of as the sum of the $\theta^3, \theta^4, \theta^5$ terms of the superfield Φ_{inv} .)

Including the moduli-dependent coefficient, the full action is then (up to an overall constant)

$$S_8 = \ln(\eta(u)) \int d^8x \frac{1}{16!} \sqrt{g} \epsilon^{\alpha_1 \dots \alpha_{16}} \left(D_{\alpha_1} \dots D_{\alpha_{16}} W^4 - 8i(\Gamma^0 \Gamma^a)_{\alpha_1 \bar{\alpha}} \psi_a^{*\bar{\alpha}} D_{\alpha_2} \dots D_{\alpha_{16}} W^4 \right) \quad (6.63)$$

This action has manifest $N = 2$ supersymmetry in 8 dimensions (after the truncation already described), and is clearly the reduction of an action with 11-dimensional Lorentz invariance. To find the explicit M-theory action, we need to evaluate the superspace derivatives (or alternatively, perform an integration over the superspace coordinates) and obtain the action in coordinate space. The resulting action can be dimensionally oxidized to eleven dimensions.

7 Discussion

We have found part of an action in eight dimensions which has 32 supersymmetries. This action encodes all terms in eight dimensions involving the curvature $R_{\mu\nu\rho\sigma}$ and the three forms $H_{\mu\nu\rho m}$. In future work, we will uplift this action to obtain all the bosonic terms in the eight-derivative M-theory effective action. It should also be possible to use our technique to find the terms bilinear in fermions.

In addition to finding the explicit action, there are several directions of interest to pursue.

Knowledge of the M-theory action allows us to find the one-loop type IIA action by a dimensional reduction. It would be interesting to develop techniques to fix the tree level part of the type IIA action as well. Similarly, we would like to work out the action for M-theory compactified on arbitrary tori.

More speculatively, we may be finding hints about the *off-shell* superspace formulation of the theory. Little is known currently about the off-shell superspace formulation of theories with 32 supercharges; even the auxiliary field content has not been determined. Our results here suggest that if such a formulation exists, it should exist in eight dimensions rather than eleven. It may be that to obtain a manifestly supersymmetric formulation, we have to give up manifest Lorentz invariance. It would be very interesting to see if our results can be extended to make this explicit; understanding the structure of the fermion bilinears will also help in this.

Acknowledgements: We are grateful to N. Berkovits, S. Deser, S. de Haro, M. Green, P. Howe, A. Sinkovics and K. Skenderis for useful comments.

The author is supported in part by NSF Grant PHY-0354993.

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